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DISSIPATIVE EFFECTS ON MHD STAGNATION POINT NANO-FLUID FLOW PAST A STRETCHABLE SURFACE WITH MELTING

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Abstract

Steady flow of a viscous, incompressible, electrically conducting, chemically reacting and heat radiating nano-fluid over a stretching sheet, with melting, in the presence of an applied transverse magnetic field considering viscous and joule dissipations into account is investigated. Using adequate similarity transformations non-linear partial differential equations, governing to the problem, are transformed into non-linear ordinary differential equations and then solved using bvp4c (Matlab's boundary value problem solver). A comparison is done to validate the numerical results obtained in the present paper with the existing literature. Numerical results of velocity

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$f'(\eta)$, temperature $\theta(\eta)$, and species concentration $\phi(\eta)$ are depicted graphically. Numerical values of skin friction coefficient $-f''(0)$, local Nusselt number $-\theta'(0)$, local Sherwood number $-\phi'(0)$ are presented in tabular form. Such nano-fluid flows find applications in heat transfer processes, pharmaceutical processes, domestic refrigerators, heat exchanger, engine cooling, chiller, vehicle thermal management etc.

1. Introduction

Nano-fluids have attracted researchers due to their varied and wide applications in industries, such as, engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, nuclear reactor coolant etc. To refer to the fluid with suspended nanoparticles, the term "*nano-fluid*" has been foremost introduced by Choi and Eastman [1]. These fluids have unique thermo-physical properties. Choi et al. [2] encountered that the thermal conductivity of the base fluids (conventional fluids) is being enhanced (around 40% to 150%) significantly by mixing a small amount (<1% volume fraction) of nanoparticles in the base fluid. To examine different aspects of the problem, several researchers [3-5] carried out research studies involving the flow of nano-fluids. It was Crane [6] who first investigated the flow of fluid over a linearly stretching sheet. This problem is of particular interest since an exact solution of the two-dimensional Navier-Stokes equations has been obtained by him. After his work, the fluid flow past a stretching surface has attracted researchers and sufficient amount of work has been carried out [7-9]. Heat transfer over stretching/shrinking surface has its own significance due to its wide applications in industrial and manufacturing processes. Many researchers [10-12] studied fluid flow problems on heat transfer over stretching/shrinking surfaces.

Viscous and Joule dissipation effects are important in a number of fluid engineering devices. The energy dissipated due to motion of the fluid and retardation due to the application of magnetic field into the system, has a heating/cooling effect on the surface which result in significant heat transfer to the fluid in the boundary layer region. The hydromagnetic nanofluid flow due to a stretching/shrinking sheet with viscous dissipation and chemical reaction effects was studied by Kameswaran et al. [13]. To describe the nano-fluid flow, they used the model "nanoparticle volume fraction". Hady et al. [14] investigated radiation effect on the viscous flow of a nanofluid and heat transfer over a nonlinearly stretching sheet. Khan et al. [15] studied unsteady natural convection boundary layer flow of a viscous, incompressible nano-fluid along a stretching sheet with thermal radiation and viscous dissipation effects in the presence of an external transverse magnetic field. There are several industrial situations where the surface is convectively heated by external source. This causes a certain change in the surface temperature gradient and affects the temperature of the fluid within the boundary layer. Makinde and Aziz [16] investigated the boundary layer flow induced in a nanofluid due to a linearly stretching sheet. In their governing transport equation, they included Brownian motion and thermophoresis effects. Recently, Mahatha et al. [17] investigated dissipative effects in boundary layer MHD nanofluid flow past a stretchable sheet with Newtonian heating.

Accompanied with melting (or solidification) characteristics, the heat transfer has wide utilization in the area of melting of permafrost, solidification of magma, and in the process of silicon wafer [18]. Epstein and Cho [19] discussed the usefulness of melting phenomenon in the laminar flow over flat surface. Gireesha et al. [20] studied the stretched flow of viscous nano-liquid with stagnation point, considering melting heat transfer and inclined MHD into the problem. Hayat et al. [21] scrutinized the effect of melting parameter in the flow of a chemically reacting fluid. A comprehensive study of heat transfer phenomenon in nano-fluid flow was made by many researchers [22-24]. Recently, Ibrahim [25] studied the melting effects and heat transfer of a nano-fluid past a stretching sheet.

Though the considerable amount of work has been done in the nano-fluid flow over a stretching surface, still more attention is needed to study the effects of viscous and joule dissipations on melting heat transfer of a nano-fluid flow past a stretching sheet. Hence authors were motivated to investigate the effects of viscous and joule dissipations, and melting heat transfer on MHD nano-fluid flow over a stretchable surface. Objective of the present paper is to investigate the steady flow of a viscous, incompressible, electrically conducting nano-fluid over a stretching sheet, with melting, in the presence of an applied transverse magnetic field taking viscous and joule dissipations into account.

2. Mathematical Model of the Problem

Consider a steady two-dimensional flow and heat transfer of a viscous, incompressible, electrically conducting, nano-fluid past a stretchable sheet. The sheet is melting steadily. Transverse magnetic field (parallel to y-axis) is applied to the flow which is of strength $B = B_0$. A coordinate system has chosen such a way that x-axis is extending along the stretching sheet and y-axis normal to it (see Figure 1). Temperature of the sheet is T_m, concentration C take constant value C_w. The ambient value attained as y tends to infinity of T and C are denoted by T_∞ and C_∞, respectively, where $T_\infty > T_m$. Free stream velocity takes the form $U_\infty = bx$ and velocity of the sheet is $u_w = ax$, where a and b are positive constants. Figure 1 represents geometrical model of the flow.

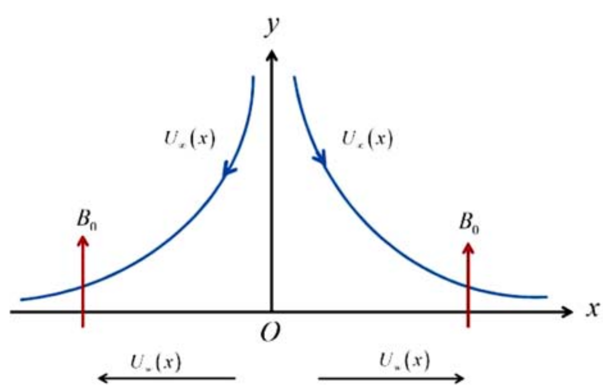

Figure 1. Geometrical model of the flow.

Under the assumptions made above boundary layer equations, governing to the present flow problem, are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + U_\infty\frac{\partial U_\infty}{\partial x} + \frac{\sigma B_0{}^2}{\rho_f}\left(U_\infty - u\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}$$
$$+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0{}^2}{\rho c_p}\left(U_\infty - u\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where

$$\alpha = \frac{k}{(\rho c)_f} \quad \text{and} \quad \tau = \frac{(\rho c)_p}{(\rho c)_f}.$$

Boundary conditions, applicable to the present problem, are:

$$\left.\begin{array}{r} u = u_w = ax,\ v = 0,\ T = T_m,\ C = C_w \text{ at } y = 0 \\ u \to U_\infty = bx,\ v = 0,\ T \to T_\infty,\ C \to C_\infty \text{ as } y \to \infty \\ \alpha\left(\dfrac{\partial T}{\partial y}\right)_{y=0} = \rho\left[\lambda + C_s\left(T_m - T_0\right)\right]v(x,0) \end{array}\right\}, \tag{5}$$

where u and v are the components of velocity along the x and y axes, respectively. Furthermore, υ, σ, ρ_f, ρ_p, α, k, $(\rho c)_f$, $(\rho c)_p$, λ and C_s are, respectively, the kinematic viscosity coefficient, electric conductivity, density of base fluid, density of nanoparticle, thermal diffusivity, thermal conductivity, heat capacity of the base fluid, heat capacity of the nanoparticle material, latent heat of the fluid, and heat capacity of the solid surface.

Introducing following similarity and dimensionless variables:

$$\left.\begin{aligned} &u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x},\ \eta = y\sqrt{\frac{a}{\upsilon}},\ \psi = \sqrt{a\upsilon}\ x\ f(\eta) \\ &\theta(\eta) = \frac{T - T_m}{T_\infty - T_m},\ \phi(\eta) = \frac{C - C_w}{C_\infty - C_w} \end{aligned}\right\}. \tag{6}$$

Equation of continuity (1) is satisfied, identically, on using (6). Further, equations (2), (3) and (4) along with boundary conditions (5) take the following forms:

$$f''' + ff'' - f'^2 + A^2 + M(A - f') = 0, \tag{7}$$

$$\theta'' + \Pr\left\{ f\theta' + Nb\ \phi'\theta' + Nt\theta'^2 + Ecf''^2 + MEc(A - f')^2 \right\} = 0, \tag{8}$$

$$\phi'' + Le\ f\phi' + \frac{Nt}{Nb}\theta'' = 0. \tag{9}$$

Boundary conditions are:

$$\left.\begin{aligned} &f'(0) = 1,\ B\theta'(0) + \Pr\ f(0) = 0,\ \theta(0) = 0,\ \phi(0) = 0,\ \text{at}\ \eta = 0, \\ &f'(\infty) \to A,\ \theta(\infty) \to 1,\ \phi(\infty) \to 1,\ \text{as}\ \eta \to \infty \end{aligned}\right\}. \tag{10}$$

Here the non-dimensional governing parameters are defined by:

$$\left.\begin{aligned} &M = \frac{\sigma B_0^2}{\rho_f a},\ Le = \frac{\upsilon}{D_B},\ \Pr = \frac{\upsilon}{\alpha},\ A = \frac{b}{a}, \\ &Nb = \frac{(\rho c)_p D_B (C_\infty - C_w)}{(\rho c)_f \upsilon},\ Nt = \frac{(\rho c)_p D_T (T_\infty - T_m)}{(\rho c)_f \upsilon T_\infty}, \\ &B = \frac{C_f (T_\infty - T_m)}{\lambda + C_s (T_m - T_0)},\ Ec = \frac{a^2 x^2}{C_p (T_\infty - T_m)} = \frac{u_w^2}{C_p \Delta T}, \end{aligned}\right\}, \tag{11}$$

where f', θ and ϕ are velocity, temperature and concentration, respectively, in non-dimensional form. M, Le, Pr, A, Nb, Nt and Ec are, respectively, the magnetic parameter, Lewis number, Prandtl number, velocity ratio parameter, Brownian diffusion coefficient, thermophoretic diffusion coefficient, and Eckert number. Dimensionless melting parameter (B) is the combination of Stefan numbers $\frac{C_f(T_\infty - T_m)}{\lambda}$ (for liquid phase) and $\frac{C_s(T - T_0)}{\lambda}$ (for solid phase).

From the engineering point of view, physical quantities which are of much interest are skin friction coefficient C_f, the local Nusselt number Nu_x and the local Sherwood number Sh_x are defined as:

$$C_f = \frac{\tau_w}{\rho u_w^2},\ Nu_x = \frac{xq_w}{k(T_\infty - T_m)},\ Sh_x = \frac{xh_m}{D_B(C_\infty - C_w)}, \tag{12}$$

where the wall shear stress τ_w, the wall heat flux q_w and wall mass flux h_m, are given by

$$\tau_w = \mu\frac{\partial u}{\partial y},\ q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0},\ h_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}. \tag{13}$$

Using equation (13) and similarity variables, we obtain

$$C_f\sqrt{\mathrm{Re}_x} = -f''(0),\ \frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = -\theta'(0),\ \frac{Sh_x}{\sqrt{\mathrm{Re}_x}} = -\phi'(0), \tag{14}$$

where Re_x is local Reynolds number.

3. Numerical Procedure and Validation

The non-linear equations (7)-(9) with boundary conditions (10) have been solved by using bvp4c routine of MATLAB. To ensure accuracy, consistency and reliability of the results obtained numerically, a comparison is done for the values of coefficient of skin friction $-f''(0)$, Nusselt number $-\theta'(0)$ Sherwood number $-\phi'(0)$, obtained by Ibrahim [25] with the respective values calculated in the present paper. This comparison is presented in Table 1 and it has been found that the present results and the results obtained by Ibrahim [25] are in good agreement.

Table 1. Computations showing comparison with Ibrahim [25] for $Nb = Nt = 0.5,\ \Pr = 1,\ Le = 2,\ Ec = 0$

			Ibrahim [25]			Present paper		
A	M	B	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1			−1.1923	−0.5954	−0.2060	-1.19	0.6	0.21
0.2			−1.0914	−0.6405	−0.2228	-1.09	0.64	0.22
0.3			−0.9825	−0.6786	−0.2372	-0.98	0.68	0.24
0.6			−0.6070	−0.7728	−0.2731	-0.61	0.77	0.27
1.5			0.91	−0.9861	−0.3544	0.91	0.99	0.35
2			1.9687	−1.0835	−0.3913	1.97	1.08	0.39
2.5			3.1604	−1.1720	−0.4248	3.16	1.17	0.42
0.5	1		−0.7401	−0.7437	−0.2620	-0.74	0.74	0.26
	2		−0.8798	−0.7321	−0.2574	-0.88	0.73	0.26
	3		−1.0019	−0.7234	−0.2540	-1	0.72	0.25
	1	0.1	−0.7279	−1.0063	−0.3145	-0.73	1.01	0.31
		0.5	−0.6608	−0.7513	−0.2650	-0.66	0.75	0.27
		1	−0.6142	−0.5834	−0.2307	-0.61	0.58	0.23
		10	−0.4596	−0.1429	−0.0972	-0.46	0.14	0.1

4. Results and Discussion

Keeping in view to analyze the effects of different parameters viz. velocity ratio parameter A, magnetic parameter M, thermal diffusion parameter Pr, Brownian motion parameter Nb, thermophoresis parameter Nt, Lewis number Le, melting parameter B and Eckert number Ec, the profiles of nano-fluid velocity, nano-fluid temperature and nano-particle concentration are depicted graphically in Figures 2-25. Also the numerical values of coefficient of skin friction $-f''(0)$, Nusselt number $-\theta'(0)$ and Sherwood number $-\phi'(0)$ are computed and tabulated in Table 2.

Figures 2 to 9 represent the effects of velocity ratio, magnetic field, thermal diffusion, Brownian motion, thermophoresis effect, Lewis number, melting parameter and the Eckert number on nano-fluid velocity. It is observed from Figures 2 to 9 that the fluid velocity is an increasing function of velocity ratio, Brownian diffusion, themophoretic diffusion, Lewis number, melting parameter, Eckert number while it is a decreasing function of parameter M and $\Pr$. This implies that the fluid velocity is getting enhanced by velocity ratio, Brownian diffusion, thrmophoretic diffusion, Lewis number, melting parameter, Eckert number/viscous dissipation. Prandtl number represents the strength of thermal diffusion. As we know thermal diffusion decreases on increasing Prandtl number. Thus, we can conclude that the thermal diffusion has also the tendency to enhance nanofluid velocity. Magnetic field is having a tendency to reduce nanofluid velocity.

Figures 10-17 represent the effects of various parameters on the fluid temperature. It is revealed from Figures 10-17 that the nano-fluid temperature is getting increased with the increase in A, Pr, Nb and Nt while it is getting decreased with the increase in M and B. With the increase in Le and Ec the nanofluid temperature is getting increased near the plate. This implies that velocity ratio, Brownian diffusion and Thermophoretic diffusion enhance the nanofluid temperature while magnetic field, thermal diffusion and melting of the sheet reduce the nanofluid temperature. In addition to it Lewis number and viscous dissipation have the tendency to induce nanofluid temperature in the boundary layer region. These two parameters have reverse effect on the fluid temperature outside the boundary layer region.

Figures 18 to 25 represent the effects of various parameters on the fluid concentration. It is evident from Figures 18-25 that the nanofluid concentration is getting increased on increasing A while it is getting decreased on increasing M, Nt, B and Ec. Very near to the plate, fluid concentration is getting decreased with the increase in Pr, Nb and Le. This shows that velocity ratio has the tendency to enhance the fluid concentration while magnetic field, thermophoretic diffusion, melting of the sheet and viscous dissipation have reverse effect on it. In the boundary layer region, thermal diffusion causes enhancement in fluid concentration while Brownian diffusion and Lewis number have reverse impact on it.

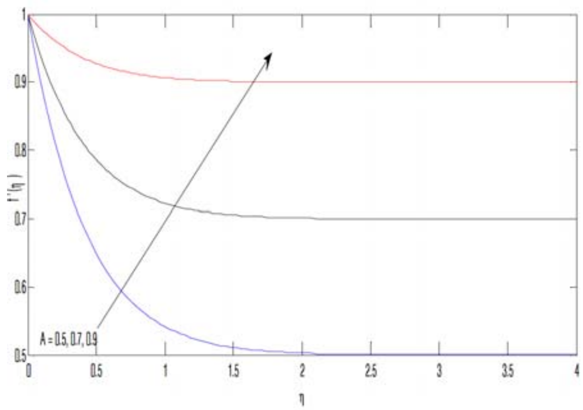

Figure 2. Velocity profiles for *A*.

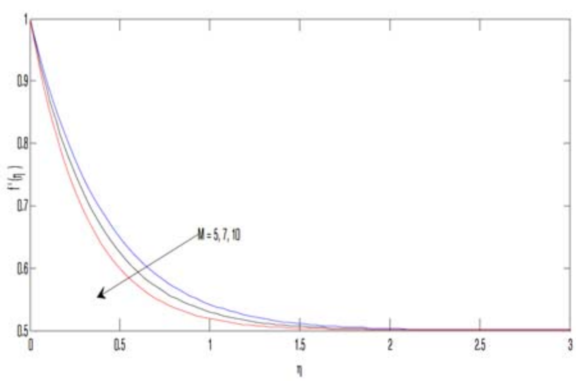

Figure 3. Velocity profiles for *M*.

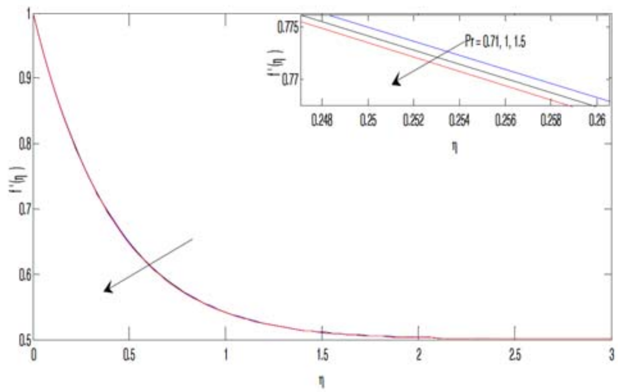

Figure 4. Velocity profiles for Pr.

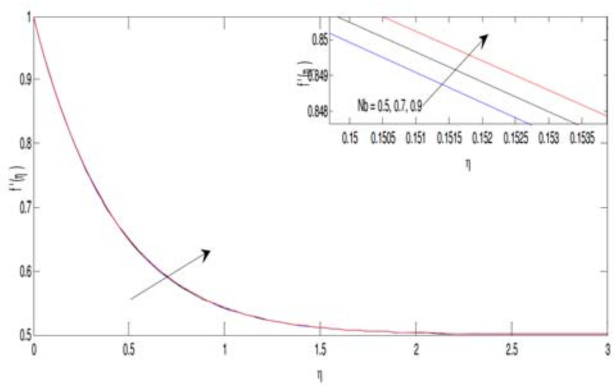

Figure 5. Velocity profiles for *Nb*.

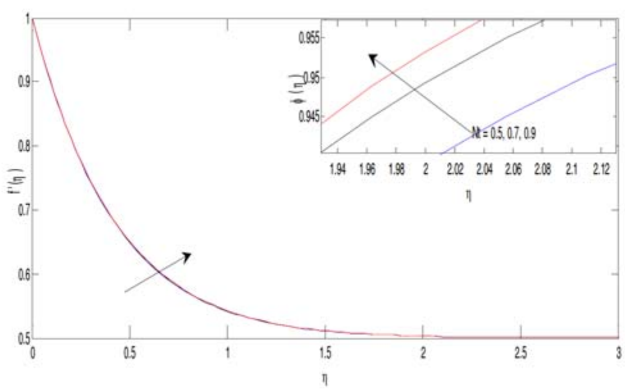

Figure 6. Velocity profiles for *Nt*.

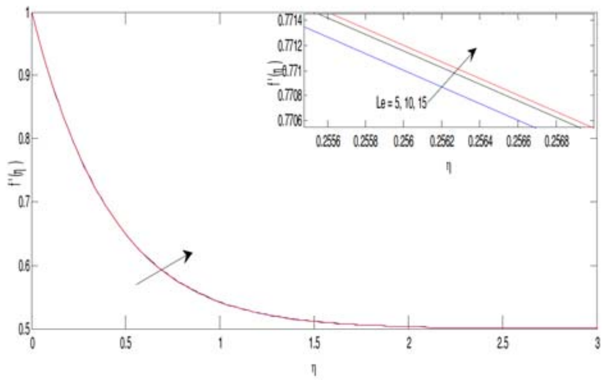

Figure 7. Velocity profiles for *Le*.

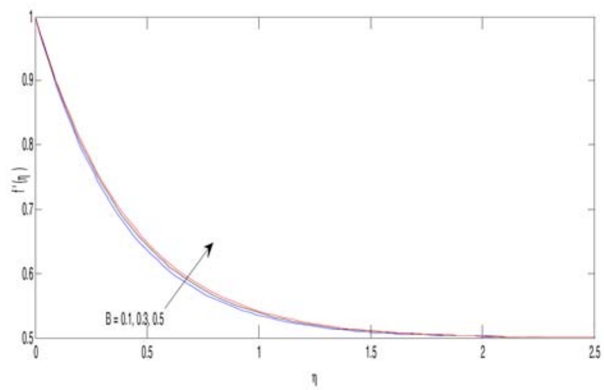

Figure 8. Velocity profiles for *B*.

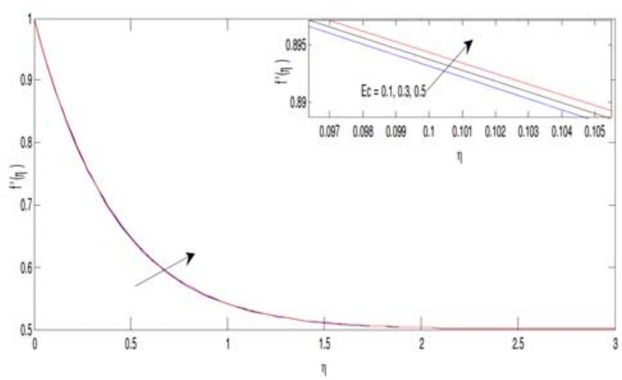

Figure 9. Velocity profile for *Ec*.

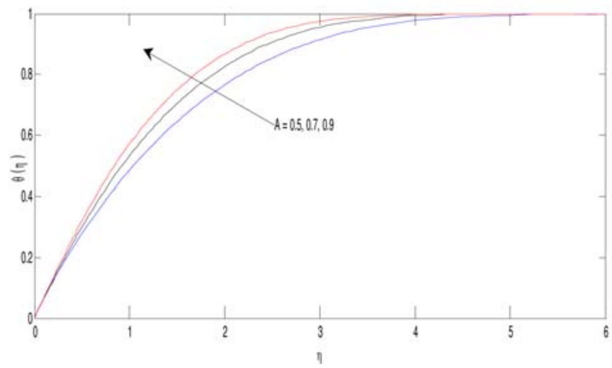

Figure 10. Temperature profiles for *A*.

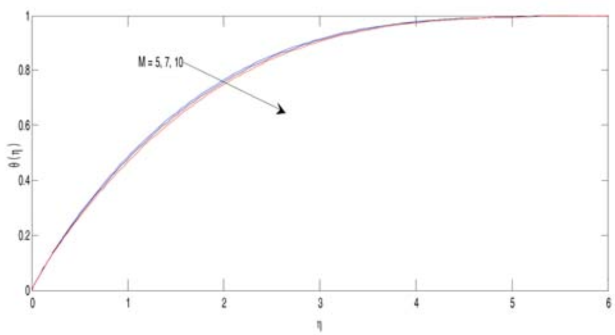

Figure 11. Temperature profiles for *M*.

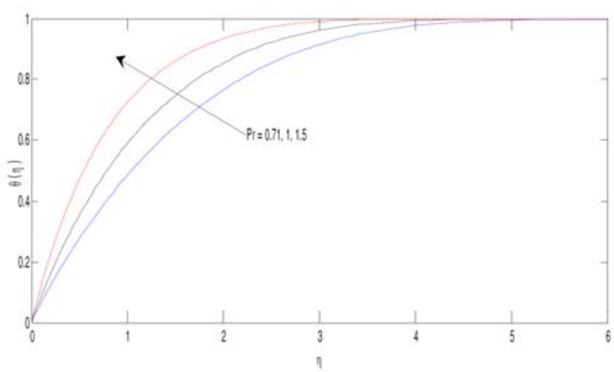

Figure 12. Temperature profiles for Pr.

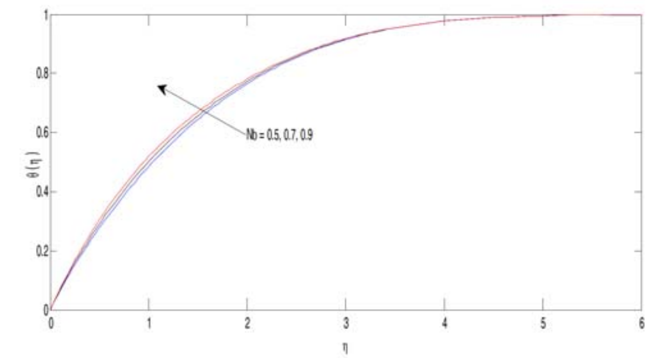

Figure 13. Temperature profiles for *Nb*.

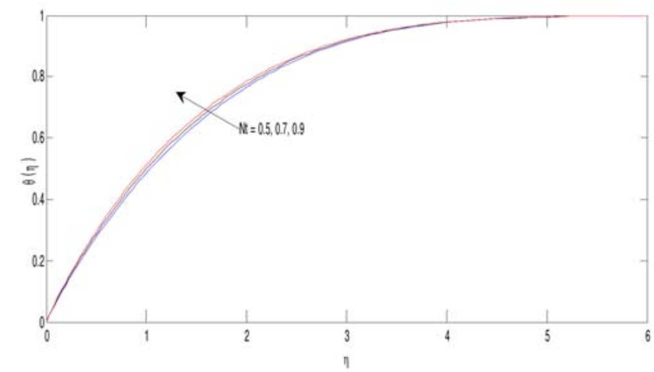

Figure 14. Temperature profiles *Nt*.

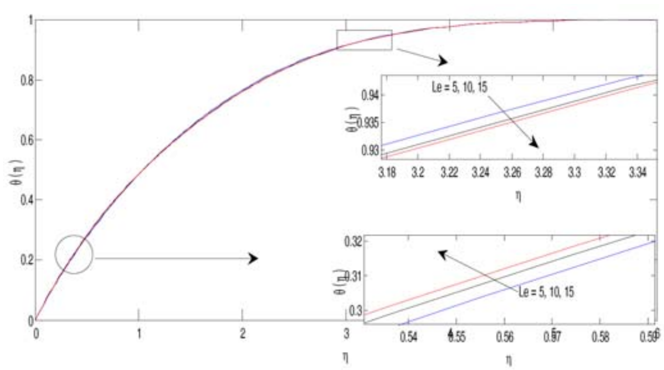

Figure 15. Temperature profiles for *Le*.

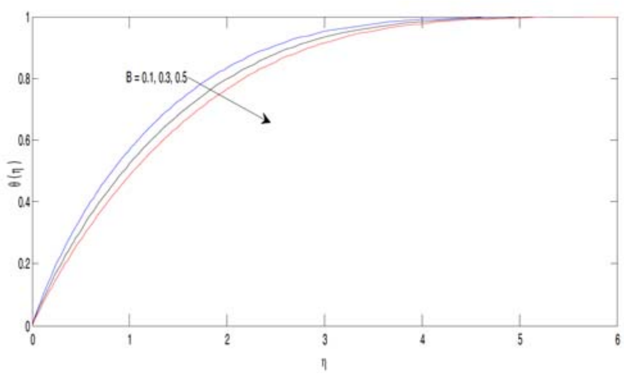

Figure 16. Temperature profiles for *B*.

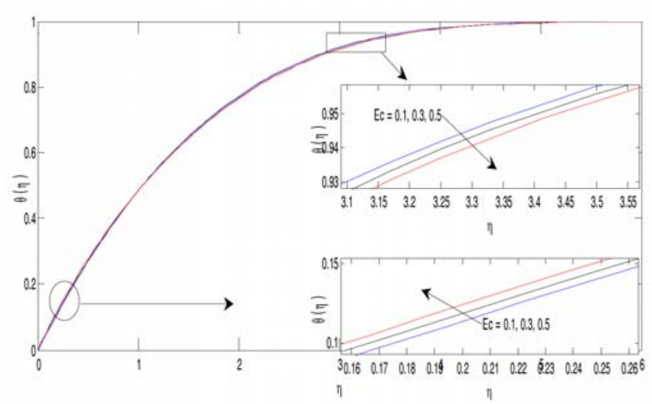

Figure 17. Temperature profiles for *Ec*.

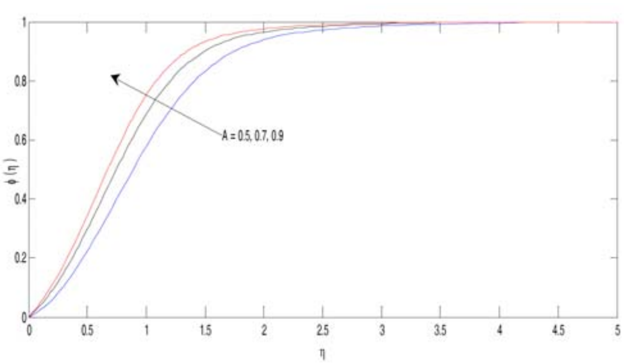

Figure 18. Concentration profiles for *A*.

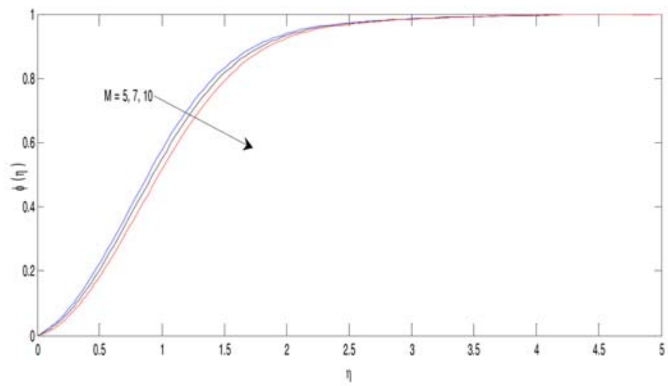

Figure 19. Concentration profiles for *M*.

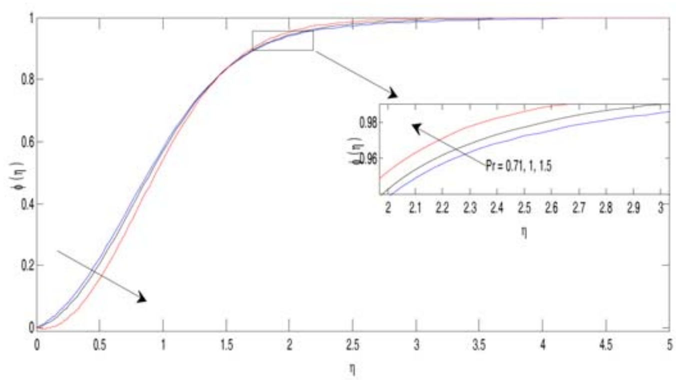

Figure 20. Concentration profiles for Pr.

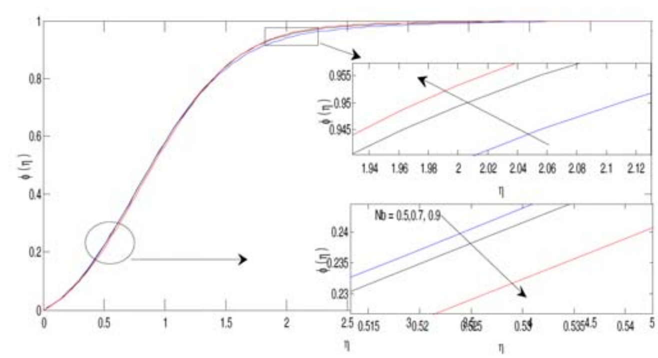

Figure 21. Concentration profiles for *Nb*.

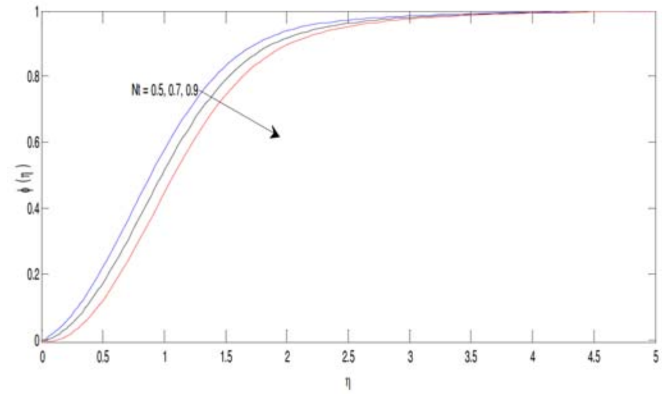

Figure 22. Concentration profiles for *Nt*

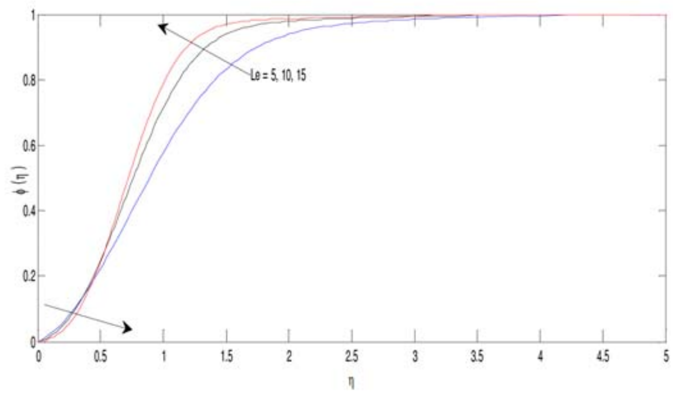

Figure 23. Concentration profiles for *Le*.

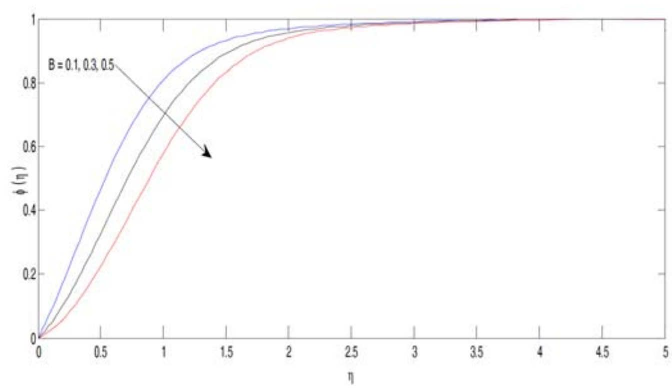

Figure 24. Concentration profiles for *B*.

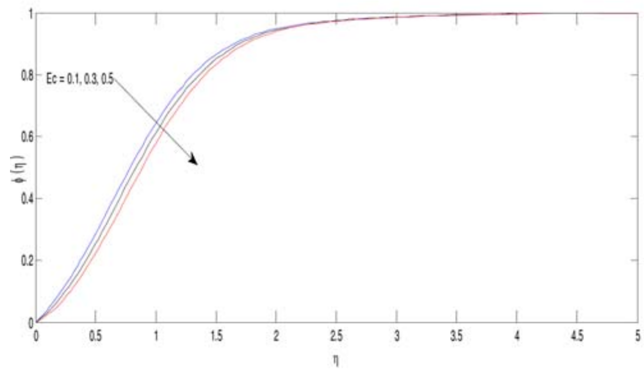

Figure 25. Concentration profiles for *Ec*.

Table 2 represents the effects of various parameters on the co-efficient of skin friction, Nusselt number and Sherwood number. It is observed from the table that on increasing M and Pr the numerical values of skin friction is getting increased while these values are getting decreased on increasing A, Nb, Nt, Le, B and Ec. The numerical values of Nusselt number is getting increased with the increase in M, Pr, Nb, Nt, Le and Ec while these values are getting decreased with the increase in B. Also, these values shows oscillatory in nature with respect to the values of A. The numerical values of Sherwood number are increasing with the increase in A. While these values are decreasing with the increase in M, Le, B and Ec. These values show oscillatory in nature with respect to the values of Pr, Nb and Nt. This implies that magnetic field induces the skin friction co-efficients while all other parameters such as velocity ratio, thermal diffusion, Brownian diffusion, thermophoretic diffusion, Lewis number, melting of sheet and viscous dissipation have reverse effect on it. The rate of heat transfer is induced by magnetic field, Brownian diffusion, thermophoretic diffusion, Lewis number and viscous dissipation while it is reversely affected by thermal diffusion and melting of sheet. It has an oscillatory nature for velocity ratio parameter. The velocity ratio causes for the enhancement in the rate of mass transfer while magnetic field, Lewis number, melting of the sheet and viscous dissipation are the cause for the reduction in rate of mass transfer. The rate of mass transfer shows oscillatory in nature for thermal diffusion, Brownian diffusion and thermophoretic diffusion.

Table 2. Effects of various parameters on coefficient of skin-friction, Nusselt number and Sherwood numbers

A	M	Pr	Nb	Nt	Le	B	Ec	$-C_f\sqrt{\mathrm{Re}_x}$	$\frac{Nu_x}{\sqrt{\mathrm{Re}_x}}$	$-\frac{Sh_x}{\sqrt{\mathrm{Re}_x}}$
0.5								-1.18129	0.692702	0.170759
0.7								-0.72767	0.671328	0.335036
0.9								-0.24768	0.68243	0.424127
	5							-1.18129	0.692702	0.170759
	7							-1.35795	0.710431	0.116841
	10							-1.58839	0.734676	0.04964
		0.71						-1.18129	0.692702	0.170759
		1						-1.18735	0.923389	0.090518
		1.5						-1.19209	1.324053	-0.112717
			0.5					-1.18129	0.692702	0.170759
			0.7					-1.1758	0.726495	0.188619
			0.9					-1.17042	0.759777	0.186373
				0.5				-1.18129	0.692702	0.170759
				0.7				-1.17711	0.71844	0.049437
				0.9				-1.17275	0.745344	-0.078581
					5			-1.18129	0.692702	0.170759
					10			-1.18019	0.699494	0.096532
					15			-1.17986	0.701483	0.032361
						0.1		-1.26662	0.936865	0.717497
						0.3		-1.21724	0.792299	0.359173
						0.5		-1.18129	0.692702	0.170759
							0.1	-1.19819	0.589703	0.363186
							0.3	-1.18969	0.641345	0.263573
							0.5	-1.18129	0.692702	0.170759

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